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SOLUTION OF THE QUARTIC EQUATION,

$$x^4+Ax+B=0.$$

By MR. A. M. SAWIN, Evansville, Wis.

FROM Harriot's laws we easily derive the root relations

$$x=-a,\ x=-b,\ x=\tfrac{1}{2}\,[a+b\pm\sqrt{(-3a^2+2ab+3b^2)}],$$

whence

$$x^4+(a^3+a^2b+ab^2+b^3)x+(a^3b+a^2b^2+ab^3)=0,$$
$$\therefore\ A=a^3+a^2b+ab^2+b^3,$$
$$B=a^3b+a^2b^2+ab^3.$$

Put y for $a-b$ and z for $a+b$; we get

$$z^3+zy^2=2A,$$
$$3z^4-2z^2y^2-y^4=16B.$$

Eliminating y we get the cubic resolvent in the form

$$z^6-4Bz^2-A^2=0,$$

from which z can be found. We then find y from the relation

$$y^2=\frac{2A}{z}-z^2,$$

and calculating a and b we can form at once the four roots. The form of the equation is interesting from its occurrence in occasional problems in the resistance of materials and in hydraulic engineering.

[The root combination here used was employed first by Bette in an obscure dissertation (*Neue Auflösung der biquadratischen Gleichungen*, 1854). The form of resolvent obtained is that due to Descartes.—ED.]

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APPROXIMATE EXTRACTION OF ROOTS.

By MR. HENRY HEATON, Atlantic, Ia.

THIS method is based on the remark that as the n^{th} root of a number is one of its n equal factors, the average of n nearly equal factors will be an approximation to that root. For example, 2 is a first approximation to the fifth root of 40, and since

$$40=2\times2\times2\times2\times2.5,$$
$$2.1=\tfrac{1}{5}(2+2+2+2+2.5)$$

is a still closer approximation. This may be made the basis of a new approximation, and so on. More generally, if $x+h$ be the n^{th} root of R,

$$\xi=\frac{(n-1)\,x+\dfrac{R}{x^{n-1}}}{n}=x+h+(n-1)\frac{h^2}{2x},\ \text{approximately,}$$

differs from the true root by a small quantity of the second order only. Since we can always take for x the nearest whole number to the root considered, h will be a proper fraction less than $\frac{1}{2}$, and the error committed will be less than

$$\frac{n-1}{8x};$$

that is, less than $\frac{5}{4}(n-1)$ units in the m^{th} place of decimals, m being the number of integral places in x.

The principal value of the method will be found in extending the limits of application of ordinary tables of roots. For example, to find

$$\sqrt[3]{3.141593}$$

we take from a table of cubes 1.46 as the first approximation. We find

$$3.141593 \div 1.46^2 = 1.47382;$$
$$\therefore \xi = 1.46461$$

within $2\frac{1}{2}$ units of the last place. The true root is 1.46459. The next approximation would give nine places, and so on.

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MR. JAMES GLAISHER'S FACTOR TABLES AND THE DISTRIBUTION OF PRIMES.

BY PROF. W. W. JOHNSON, Annapolis, Md.

THE publication of Mr. James Glaisher's *Factor Table for the Sixth Million** marks the completion of a work of extraordinary extent, and of the highest value in the theory of numbers, to which France, Germany, and England have contributed equal portions of the tireless zeal of the skillful calculator; namely, the tables which enable us to resolve into its prime factors every number less than 9,000,000. The present volume is noteworthy also as containing, in the form of an introduction of more than 100 quarto pages, a very able memoir upon the distribution of primes, embodying in tabular form the results of profound and laborious research.

The history of the factor tables is briefly as follows: The subject of the construction of extensive factor tables was agitated to a considerable extent during the latter part of the last century. Euler published a memoir in 1774 on the mode of construction of a factor table to extend to a million, and four years earlier, Lambert, in a work containing a description of a method of forming such a table, gave a folding sheet on which were given all the simple factors

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**Factor Table for the Sixth Million, containing the least factor of every number not divisible by* 2, 3 *or* 5 *between* 5,000,000 *and* 6,000,000. By JAMES GLAISHER, F. R. S. London: *Taylor and Francis,* 1883